

For the past fifty years or so, Interval Arithmetic (see, e.g., Moore et al.’s recent book [MR2482682]) has been used as a foundation for many algorithms to solve problems such as Global Optimization and robust computation of solutions of difficult, real-valued, linear and nonlinear problems. More recently, it has also been used to solve problems with quantifiers (e.g., Ratschan’s work [MR2264421]).

For some application areas, problems are systematically modelled as first-order constraints with universally and existentially quantified variables. Since a “naive” use of interval arithmetic does not permit to capture the meaning of such expressions, Gardenes et al. devised Modal Interval Arithmetic (MIA) in the mid-eighties as an extension of it in which each interval is coupled with a quantifier (modality). Since then, and despite some striking successes in narrow domains (see the work by Nathan Hayes and the Sunfish Studio firm, for example), MIA has not enjoyed a widespread recognition so far, even among the Interval Arithmetic community. That situation could be explained by the dearth of good references in English on the subject, many of the original works being available in Spanish only.

Sainz et al.’s book “Modal Interval Analysis—New Tools for Numerical Information” is supposed to be a step in the right direction as it presents all the relevant results on the subject—together with proofs and examples of applications—by members of the research team that has been at the forefront of researches on Modal Interval Arithmetic since the beginning.

The book is organised in ten chapters: Chapter 1 succinctly presents Interval Arithmetic, and highlights through examples its shortcomings with respect to quantified expressions; Chapter 2 introduces Modal Interval Arithmetic with the semantics of predicates and set operations; Chapter 3 presents the notion of MIA interval arithmetic extension of expressions, with some examples interspersed; As with Interval Arithmetic, MIA extensions may overestimate the domains of expressions. Chapter 4 gives conditions under which the extensions are optimal; Chapter 5 defines the MIA arithmetic operators and gives formulas for their actual implementation; Chapter 6 explains how to solve linear systems of the form $Ax = b$ when considering matrices and vectors of modal intervals. The meaning of the solution with respect to quantifiers is also analysed. Several examples are provided; Chapter 7 considers the MIA extension for the case of non-monotonic functions, through enclosure by inner and outer rounding; Chapter 8 introduces new quantities called “marks” to capture the imprecision of values when considering their imperfect computer representation; Arithmetic operators for marks are presented;

Chapter 9 then considers modal intervals with marks as bounds, which is supposed to remove the implementation problem of modal interval extensions of expressions in which some variables appear with two different modalities. Lastly, Chapter 10 presents some examples of application of MIA.

On the positive side, this book is a welcome addition to the literature on Modal Interval Arithmetic as a one-stop reference. On the other hand, I cannot recommend it as an introduction on the subject because of its structure and its presentation of the relevant results. As a whole, the material seems disorganized, which makes it hard to understand. Take Chapter 1, for example: the chapter starts by presenting what are intervals and explains only much later what they are useful for; Notions are also used before being introduced (e.g., the notion of “digital bound” appears first on Page 2, while it is only defined on Page 7. The same goes for the `Out()` operator). On first reading, most chapters look like collections of definitions, lemmas and theorems, without the context that would motivate their introduction in the first place, which makes for a very difficult reading likely to discourage many prospective users of MIA.

For so recent a book, the chapter on the implementation of MIA operators looks strangely dated: the C++ library developed by the research group to which the authors belong uses an obsolete compiler, Borland C++. The authors also appear unaware of the advances in FPU architectures that permit switching the rounding direction at a fraction of the cost of past processors.

The material presented in this book is complex *per se*. In particular, Chapter 8 and 9 add yet another level of sophistication for the sole purpose of being actually able to tackle a slightly larger domain of problems. However, the presentation of this material in the book makes it even harder to understand for the newcomer. The extra complexity is particularly discouraging knowing that it has been repetitively proved by other research groups that most “unique” advantages provided by MIA can be obtained with regular Interval Arithmetic (see, e.g., Arnold Neumaier, “Computer graphics, linear interpolation, and nonstandard intervals”, 2009) with much less complexity.

As a side note, the book would have also benefited from better editing by a native English speaker, as it contains nonsensical sentences that, I suspect, come from improper translation to English.